Chapter 3
Single Workstation Factory Models

3.1.

(a) $\lambda = 5$/hr
(b) $p_0 = 0.122$ $CT_q = 6.008$ hr
(c) WIP=2.437
(d) $E[\text{\# lost jobs}] = 35.64$/day
(e) $th = 3.513$/hr
(f) $CT_s = 41.62$min

3.3.

(a)
(b)
(c)

$$p_1 = \frac{\lambda}{\mu} p_0$$

$$p_2 = \frac{\lambda^2}{2\mu^2} p_0$$

$$p_3 = \frac{\lambda^3}{4\mu^3} p_0$$

$$p_0 = \left(1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{2\mu^2} + \frac{\lambda^3}{4\mu^3}\right)^{-1}$$

(d) $E[\text{server utilization}] = 0.5 p_1 + p_2 + p_3$.
(e) $E[\text{\# lost jobs}] = \lambda\, p_3$
(f) $th = \lambda\,(1 - p_3)$

3.5.

λ	$\lambda = \mu$	$\lambda = 2\mu$	$\lambda = 3\mu$	$\lambda = 4\mu$
λ_e	$\frac{3}{4}\lambda$	$\frac{7}{15}\lambda$	$\frac{13}{40}\lambda$	$\frac{21}{85}\lambda$

3.7.

$$
\begin{aligned}
\text{State 0:} \quad & \lambda p_0 = \mu p_1 \\
\text{State 1:} \quad & (\lambda + \mu)p_1 = \mu p_2 \\
\text{State 2:} \quad & (\lambda + \mu)p_2 = \lambda p_0 + \mu p_3 \\
\text{State 3:} \quad & (\lambda + \mu)p_3 = \lambda p_1 + \mu p_4 \\
\text{State 4:} \quad & (\lambda + \mu)p_4 = \lambda p_2 + \mu p_5 \\
\text{State 5:} \quad & (\lambda + \mu)p_5 = \lambda p_3 + \mu p_6
\end{aligned}
$$

and $2\lambda < \mu$.

3.9.
For $n = 1, 2, \cdots$

$$
p_n = \frac{\lambda^n}{2^{n-1}\mu^n} p_0
$$

and

$$
p_0 = \frac{2\mu - \lambda}{2\mu + \lambda}
$$

for $\lambda < 2\mu$.

3.11.

$$
WIP_q = \frac{\lambda^2}{\mu(\mu - \lambda)}
$$

3.13.

$$
CT_q = \frac{\lambda}{\mu(\mu - \lambda)}
$$

3.15.
$CT_s = 0.5691$
$CT_q = 0.1707$
$WIP_s = 2.2152$
$WIP_q = 0.6646$
$u = 0.7753$
$E[T_s] = 0.3984$
$\lambda_e = 3.8924$

3.17.
$p_0 = 0.288, \ p_{1f} = 0.209, \ p_{1s} = 0.118, \ p_2 = 0.196, \ p_3 = 0.118, \ p_4 = 0.071.$

3.19.

$WIP_s = \sum_{n=0}^{N} np_n$

$WIP_q = \sum_{n=3}^{N} (n-2)p_n$

$CT_s = WIP_s/\lambda_e$

$CT_q = WIP_q/\lambda_e$

$E[EBS] = p_1 + 2\sum_{n=2}^{N} p_n$

$u = E[EBS]/2$

$th_s = \lambda_e = \lambda(1 - p_N)$

3.21.

A state is an ordered pair, (i,j) with the first component denoting a part going through the needs-analysis ($i = 1$) or a part going through a second analysis as required for the special-purpose analysis ($i = 2$) or the analysis is blocked due to a full system ($i = b$). The second component indicates the number of parts being processed (i.e., zero or one) plus the number in queue; thus, $j = 0, 1, 2, 3$.

$p_{10} = 0.030, \quad p_{11} = 0.069, \quad p_{12} = 0.092, \quad p_{13} = 0.146,$
$p_{20} = 0.184, \quad p_{21} = 0.071, \quad p_{22} = 0.068, \quad p_{23} = 0.052, \quad p_{b3} = 0.289$

Machine utilization $= 0.786$
Order employee utilization $= 0.711$
Throughput $= 0.674$/hr.

3.23.

$CT_s = 0.5052$ hr
$WIP_s = 1.4177$
$th = 2.8065$/hr

3.25.

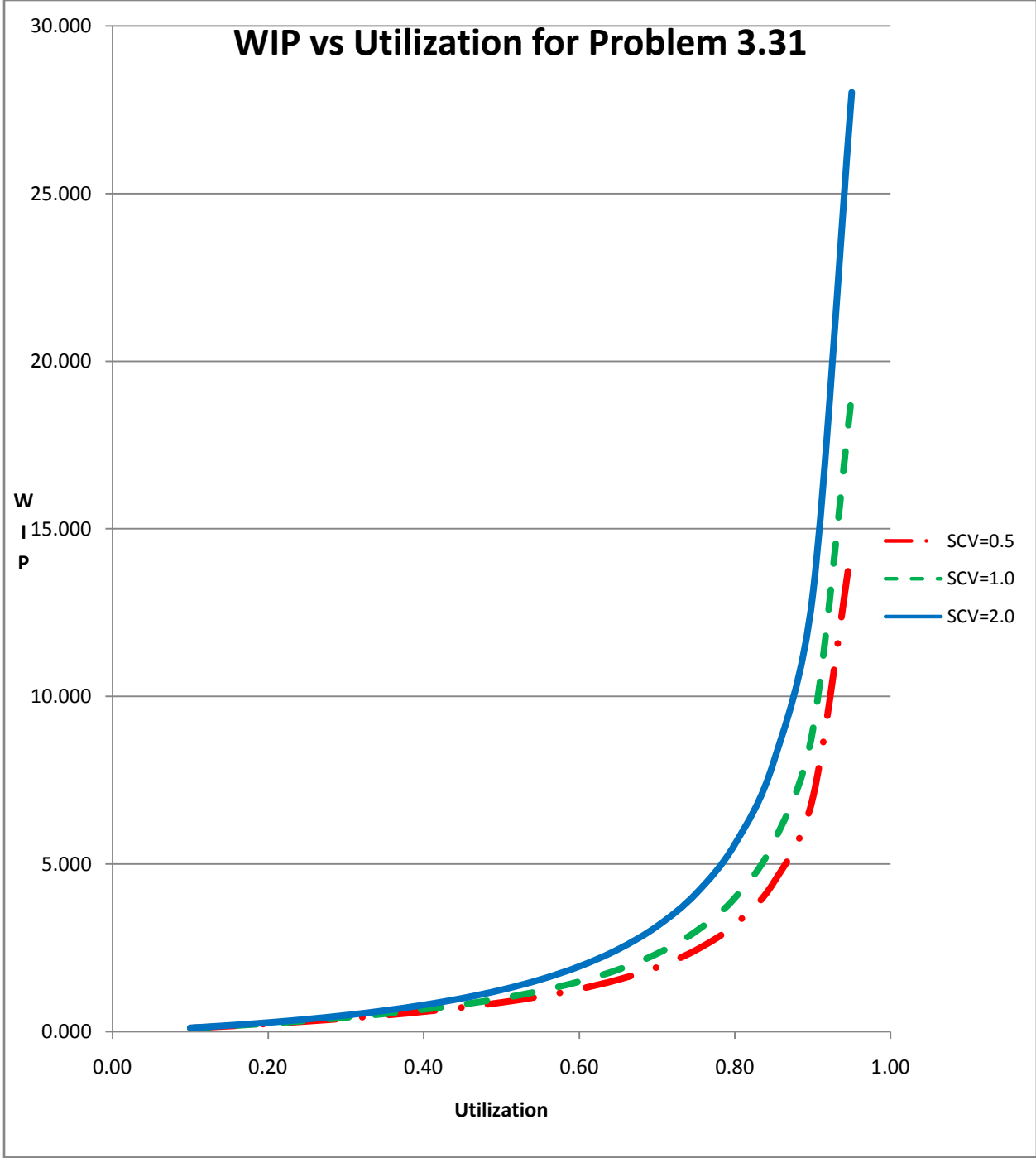| Case	$E[X]$	$C^2[X]$	λ_1	α	λ_2
i	1	5/4	2	0.4	0.8
ii	4/3	3/2	3/2	1/3	1/2
iii	5	2	0.4	0.25	0.1
iv	5/8	5/2	3.2	0.2	0.64

3.27.

$CT_s = 1.0039$ hr
$WIP_s = 1.9026$
$th = 1.8953$/hr

3.31. *Problem 3.31 in the textbook incorrectly referenced Problem 3.29; it should have referenced Problem 3.30*

WIP vs Utilization for Problem 3.31

3.33.

Because the different type jobs have different processing characteristics, all jobs in the system must be identified. Thus, a state of the system is designated by a single item, an ordered pair, or a three-tuple. The empty state is designated by 0; if one job is in the system, it is designated by (1) or (2); if two jobs are in the sytem, the state is designated as (11), (12), (21), or (22), and if three jobs are in the system, the possible states are (111), (112), (121), (122), (211), (212), (221), and (222). The equations of state are

$$(\lambda_1 + \lambda_2)p_0 = \mu_1 p_1 + \mu_2 p_2$$
$$(\lambda_1 + \lambda_2 + \mu_1)p_1 = \lambda_1 p_0 + \mu_1 p_{11} + \mu_2 p_{12}$$
$$(\lambda_1 + \lambda_2 + \mu_2)p_2 = \lambda_2 p_0 + \mu_1 p_{21} + \mu_2 p_{22}$$
$$(\lambda_1 + \lambda_2 + \mu_1)p_{11} = \lambda_1 p_1 + \mu_1 p_{111} + \mu_2 p_{112}$$
$$(\lambda_1 + \lambda_2 + \mu_1)p_{21} = \lambda_2 p_1 + \mu_1 p_{211} + \mu_2 p_{212}$$
$$(\lambda_1 + \lambda_2 + \mu_2)p_{12} = \lambda_1 p_2 + \mu_1 p_{121} + \mu_2 p_{122}$$
$$(\lambda_1 + \lambda_2 + \mu_2)p_{22} = \lambda_2 p_2 + \mu_1 p_{221} + \mu_2 p_{222}$$
$$\mu_1 p_{111} = \lambda_1 p_{11}$$
$$\mu_2 p_{112} = \lambda_1 p_{12}$$
$$\mu_1 p_{211} = \lambda_2 p_{11}$$
$$\mu_2 p_{212} = \lambda_2 p_{12}$$
$$\mu_1 p_{121} = \lambda_1 p_{21}$$
$$\mu_2 p_{122} = \lambda_1 p_{22}$$
$$\mu_1 p_{221} = \lambda_2 p_{21}$$
$$\mu_2 p_{222} = \lambda_2 p_{22}$$

Product 1
$$WIP_s(1) = p_1 + p_{12} + P_{21} + p_{122} + p_{212} + p_{221}$$
$$+ 2 \times (p_{11} + p_{112} + p211 + p_{121}) + 3p_{111}$$
$$th_s(1) = \lambda_1 \times (p_0 + p_1 + p_2 + p_{11} + p_{12} + p_{21} + p_{22})$$
$$CT_s(1) = WIP_s(1)/th_s(1)$$

Product 2
$$WIP_s(2) = p_2 + p_{12} + P_{21} + p_{112} + p_{121} + p_{211}$$
$$+ 2 \times (p_{22} + p_{122} + p212 + p_{221}) + 3p_{222}$$
$$th_s(2) = \lambda_2 \times (p_0 + p_1 + p_2 + p_{11} + p_{12} + p_{21} + p_{22})$$
$$CT_s(2) = WIP_s(2)/th_s(2)$$

All Products

$$WIP_s = WIP_s(1) + WIP_s(2)$$
$$th_s = th_s(1) + th_s(2)$$
$$CT_s = WIP_s / th_s$$