Chapter 4
Processing Time Variability

4.1.

(a) $E[T_i] = 1.5$ and $C^2[T_i] = 8$ for $i = 1, \cdots, 4$.
(b) $C^2[T] = 0.5$ yielding a 75% improvement.

4.3.

(a) $E[T] = 75$ min and $C^2[T] = 0.3777$.
(b) $E[T_{new}] = 66$ min and $C^2[T_{new}] = 0.415$ yielding a 12% improvement in mean residence time and -9.95% improvement (i.e., it increased) in SCV.

4.5.
$E[T_e] = 4.2777$ hr and $C^2[T_e] = 1.463$

4.7.
The state space for this system is

$$\{\, 0, 1, (1b), 2, (2b), 3, (3b) \,\}$$

where the state (ib) denotes that there are i jobs in the system but processing is blocked because of a breakdown.

$p_0 = 0.151, \quad p_1 = 0.189, \quad p_{1b} = 0.032, \quad p_2 = 0.237, \quad p_{2b} = 0.036,$
$p_3 = 0.301, \quad p_{3b} = 0.054.$

$WIP_s = 1.831$
$CT_s = 0.568$
$th_s = 3.225$

$\Pr\{\text{idle}\} = 0.151$
$\Pr\{\text{down}\} = 0.122$
$\Pr\{\text{processing}\} = 0.727$

4.9.
The state space for this system is

$$\{\,0,1,(1b1),(1b2),2,(2b1),(2b2),3,(3b1),(3b2),\cdots\}$$

where the state i denotes that there are i jobs in the system and the processor is working and the state (ibk) denotes that there are i jobs in the system, the processor is down, and the repair work is in Phase k.

$WIP_s = 5.4366$
$CT_s = 1.0873$
$th_s = 5$

Equations (4.3) and (4.4) yield
$E[T_e] = 0.1667$ and $C^2[T_e] = 1.2143$,
and Eq. (3.19) yields
$CT_s = 1.0893$.

4.11.
A common solution procedure (and used in previous problems) is to set $p_0 = 1$ and then solve for other probabilities in terms of the value for p_0 being 1. As additional probabilities are obtained, maintain the sum of all previous probabilities. When the sum no longer increases (i.e., all additional probabilities are essentially zero) then the final probabilities are equal to the calculated value divided by the sum. Such a scheme will not work with this system because all variable cannot be expressed in terms of p_0.

4.13.
The instructions for Problems 4.13 and 4.14 should request that state diagrams be constructed. The state space for this problem is

$$\{\,0,1,b,2,(1,b),(b,b)\,\}\ .$$

The assumption is that machines cannot break unless they are processing. Machine failures are assumed to be preemptive-resume (i.e., the job is finished after the machine has been repaired). There is no queue for this system since the capacity constraint is the same as the number of available machines. The state i represents i machines processing jobs. The state b indicates one job on a machine that is undergoing repair. The state $(1,b)$ denotes one machine processing and one machine under repair. Finally, the state (b,b) represents both machines under repair.