Chapter 9
Serial Limited Buffer Models

9.1.
The servcie can be modeled as an MGE_2 system and then evaluated with Property 9.2. The MGE_2 system is defined by

$$\boldsymbol{\alpha} = (0.25, 0.75)$$

$$G = \begin{bmatrix} -2 & 2 \\ 0 & -0.0111 \end{bmatrix}$$

yielding $E[T] = 90.125$ min with an $SCV = 0.9972$.

9.3.

(a) $\mu = 0.533$, $a = 0.50$, and $\gamma = 0.800$.
(b) $\mu = 0.571$, $a = 0.25$, and $\gamma = 0.143$.
(c) $\mu = 0.333$, $a = 0.0$, and $\gamma =$ —.

9.4.
This problem should refer to a single-server system; namely a $GE_2/GE_2/1/2$ system. In addition, when an arrival is blocked, the arrival process is halted.

9.5.
The problem is best described as a $GE_2/GE_2/1/1$ system. In other words, both the arrival process and the service process are generalized Erlang distributions where the arrival process is blocked when the system is full. There is only one server and at most one part can be in the system at one time.

The state space is given as

$$\{\,(10),(20),(11),(21),(12),(22),(b1),(b2)\,\}$$

where the first component of the ordered pair gives the state of the arrival process and the second component gives the state of the service process.

(a) $\Pr\{N = 0\} = 0.4502$ and $\Pr\{N = 1\} = 0.5498$.

(b) $\Pr\{\text{Arrivals blocked}\} = 0.2106$.
(c) $\Pr\{N = 0\} = 0.4502$, $\Pr\{N = 1\} = 0.3392$, and $\Pr\{N = 2\} = 0.2106$.

9.7.
This is a continuation of the previous problem.

The second forward pass for Workstation 1:

$$\begin{aligned} p_{d,1}^{0} &= 0.3607 \\ th(1) &= 0.7855 \end{aligned}$$

The second forward pass for Workstation 2:

$$(\alpha_2, p_2, \beta_2) = (0.9626, 0.1295, 1.8006)$$

The second backwards pass for Workstation 2:

$$\begin{aligned} p_{a,2}^{F} &= 0.2252 \\ th(2) &= 0.7746 \end{aligned}$$

The second backwards pass for Workstation 1:

$$\begin{aligned} (\mu_1, q_1, \gamma_1) &= (1.1367, 0.1084, 2.1502) \\ th(1) &= 0.7792 \end{aligned}$$

The third forward pass for Workstation 1:

$$\begin{aligned} p_{d,1}^{0} &= 0.3532 \\ th(1) &= 0.7792 \end{aligned}$$

The third forward pass for Workstation 2:

$$(\alpha_2, p_2, \beta_2) = (0.9642, 0.1198, 1.8129)$$

The third backwards pass for Workstation 2:

$$\begin{aligned} p_{a,2}^{F} &= 0.2276 \\ th(2) &= 0.7780 \end{aligned}$$

The second backwards pass for Workstation 1:

$$\begin{aligned} (\mu_1, q_1, \gamma_1) &= (1.1357, 0.1108, 2.1457) \\ th(1) &= 0.7785 \end{aligned}$$